On calibration and imaging with eigenbeams

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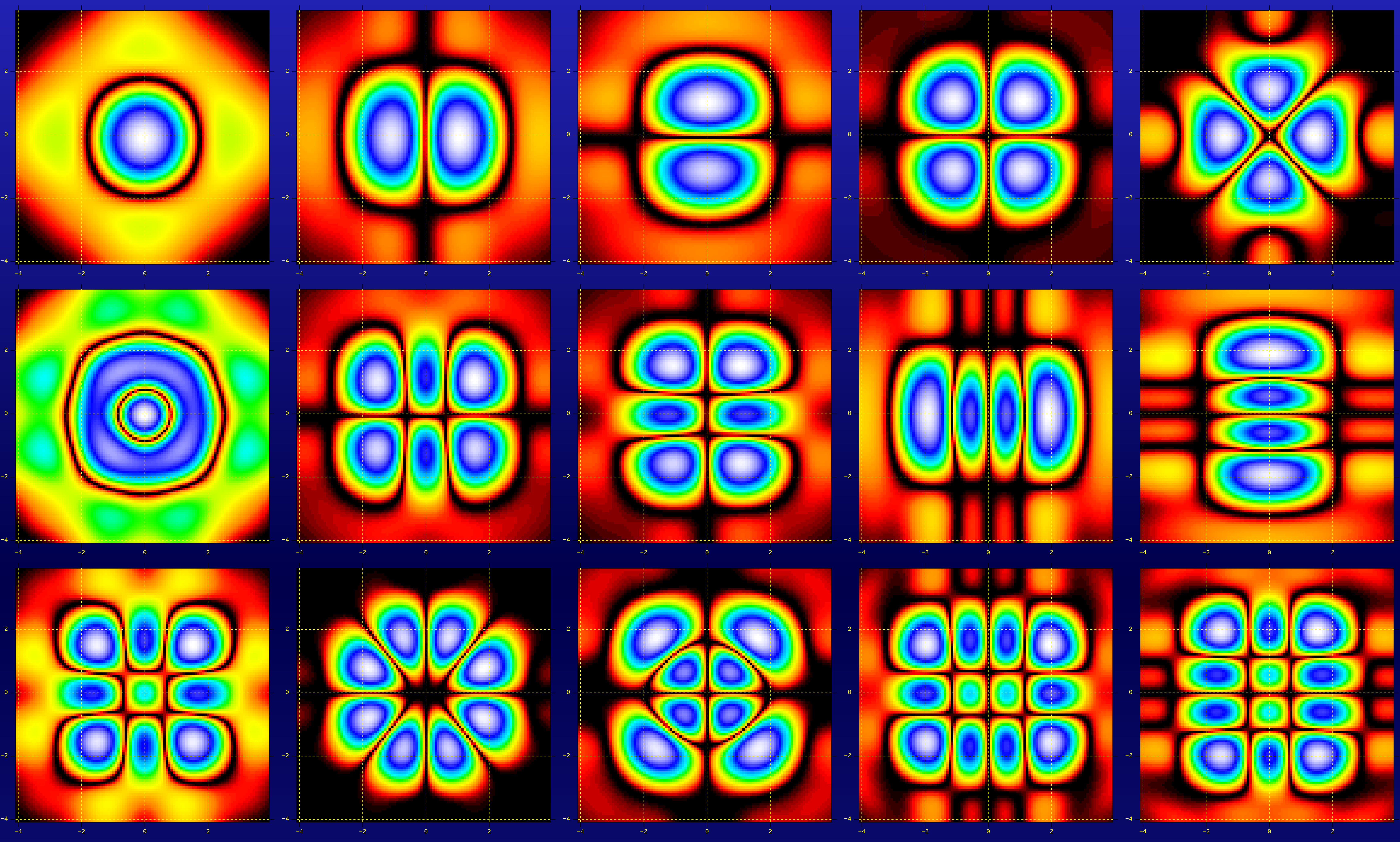

Figure 1. Eigenbeams corresponding to 15 largest eigenvalues. The simulations assumed observations at 1.4 GHz with a 15m dish and a 10×10 array of feeds with a 20 arcmin separation between elements. Offsets are given in degrees.

Introduction

Wide instantaneous field of view is one of the key science requirements for the Square Kilometre Array (SKA) as it directly affects the survey speed. A significant experience necessary to overcome the challenges of the wide field of view regime can be gained designing the SKA technology demonstrators, such as the xNTD (MIRA) and KAT. These instruments will have an array of feeds mounted in the focal plane of a dish to achieve a better performance than conventional interferometers can provide (wider instantaneous field of view and better illumination of the aperture). The unique feature of the xNTD (MIRA) approach is that array of feeds will be phased together in the beamformer. Although the number of correlator inputs per antenna is less than the number of feeds in this case, a freedom to choose weights of the linear combinations allow different optimization strategies. Below we consider an approach to non-adaptive (weights are constant) beamforming and show that only a limited number of synthetic beams is required for imaging and calibration.

What is the eigenbeam?

The beamformer calculates a linear combination of input voltages. Therefore, a synthetic voltage pattern is

$$E(\vec{s}) = \sum_l w_l E_l(\vec{s}),$$

where $E_l(\vec{s})$ is a voltage pattern of the lth element and w_l is a corresponding complex weight. The power beam is a quadratic form

$$A(\vec{w},\vec{s}) = E(\vec{s})E^*(\vec{s}) = \sum_{l,m} w_l^* E_l^* w_m E_m = \vec{w}^H \mathcal{E}(\vec{s})\vec{w},$$

where $\mathcal{E}(\vec{s})$ is a voltage pattern matrix $\mathcal{E}(\vec{s}) = \|E_l^*(\vec{s})E_m(\vec{s})\|_m^l$ for direction $\vec{s}$. The power beam can be optimized using the linear algebra. One of the possible ways is to maximize a function of weight

$$F(\vec{w}) = \int A(\vec{w},\vec{s})K(\vec{s})\,d\vec{s},$$

where $K(\vec{s})$ is an arbitrary kernel, which essentially defines the desired optimization. Substitution gives

$$F(\vec{w}) = \vec{w}^H\left[\int \mathcal{E}(\vec{s})K(\vec{s})\,d\vec{s}\right]\vec{w} = \vec{w}^H \mathcal{E}\vec{w},$$

where the matrix $\mathcal{E} = [\cdots]$ does not depend on the direction.

- This quadratic form attains its maximum under condition of $\vec{w}^H\vec{w} = 1$, if $\vec{w}$ is an eigenvector of $\mathcal{E}$ corresponding to the largest eigenvalue.
- If the beamformer has multiple outputs (i.e. the observations can be performed with a number of different weight vectors simultaneously), eigenvectors corresponding to the appropriate number of the largest eigenvalues can be used.

Possible optimizations

The choice of the kernel $K(\vec{s})$ determines the optimization.

- $K(\vec{s})$ could be a model intensity of the sky. This maximizes the power collected from all known sources in the field of view. Good approach for calibration.
- $K(\vec{s})$ could be a circular Gaussian with the width comparable to the field of view, or even a constant. This could be better for imaging.

In both cases the eigenvalue spectrum falls rather steeply (Figure 2). Therefore, a small number of eigenbeams contains most of the information.

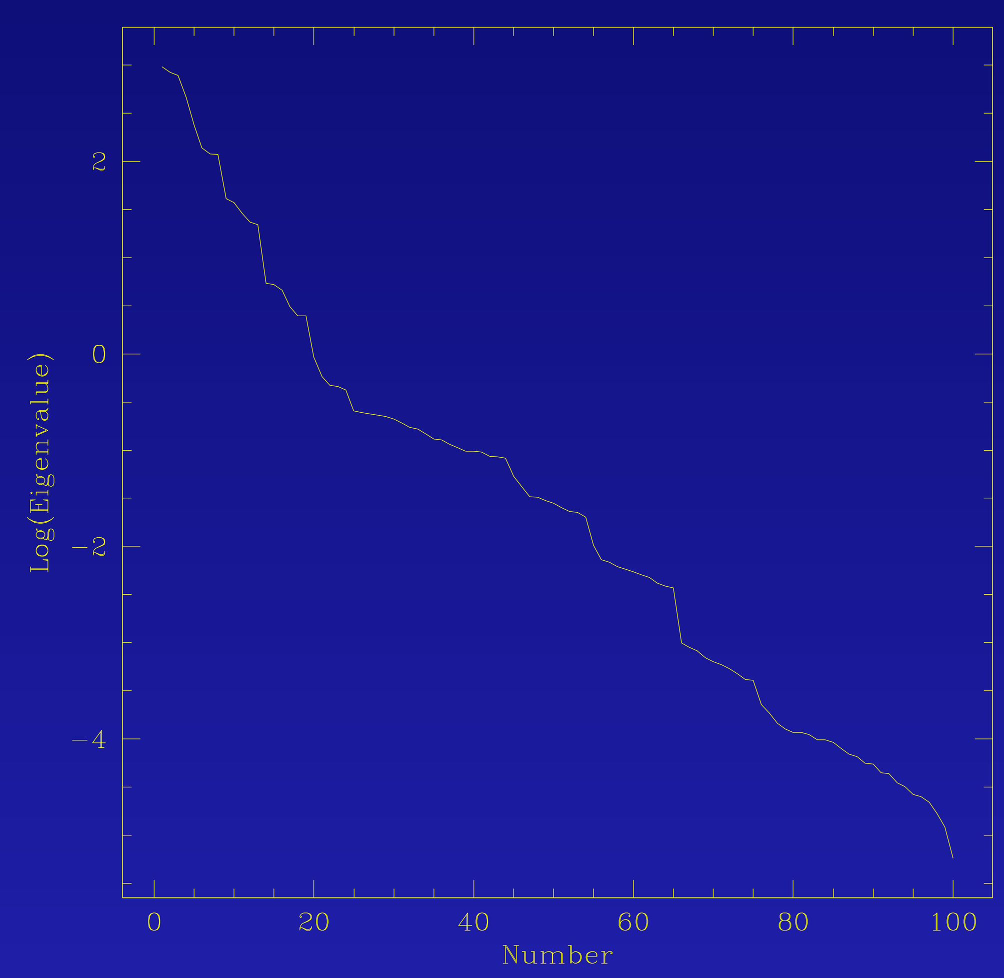

Figure 2. The eigenvalue spectrum plotted in the logarithmic scale.

Conclusions

- A relatively small number (about 10) of synthetic beams (linear combinations in the beamformer) contains most of the information.
- Only a small number of linear combinations of gains can be determined in the full-beam self-calibration procedure (using all known sources in the field of view). Therefore, one needs to track the relative gains somehow.

More details in ATNF SKA Memo 12

http://www.atnf.csiro.au/SKA/Memoseries.html