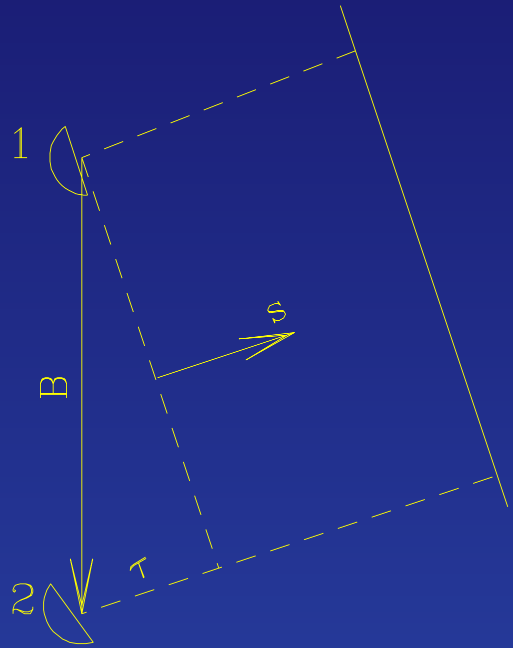
Non-Inertial Reference Frame and Dynamic Range Limit

Maxim Voronkov

(a part of the bigger project conducted in collaboration with Mark Wieringa and Tim Cornwell)

Australia Telescope National Facility



$$\overrightarrow{s} = \overrightarrow{s_0} + \overrightarrow{\delta s}$$

$$\overrightarrow{B} = \overrightarrow{R_2} - \overrightarrow{R_1}$$

Basics of interferometry:

- Monochromatic plane wave, single polarization, no primary beam effects

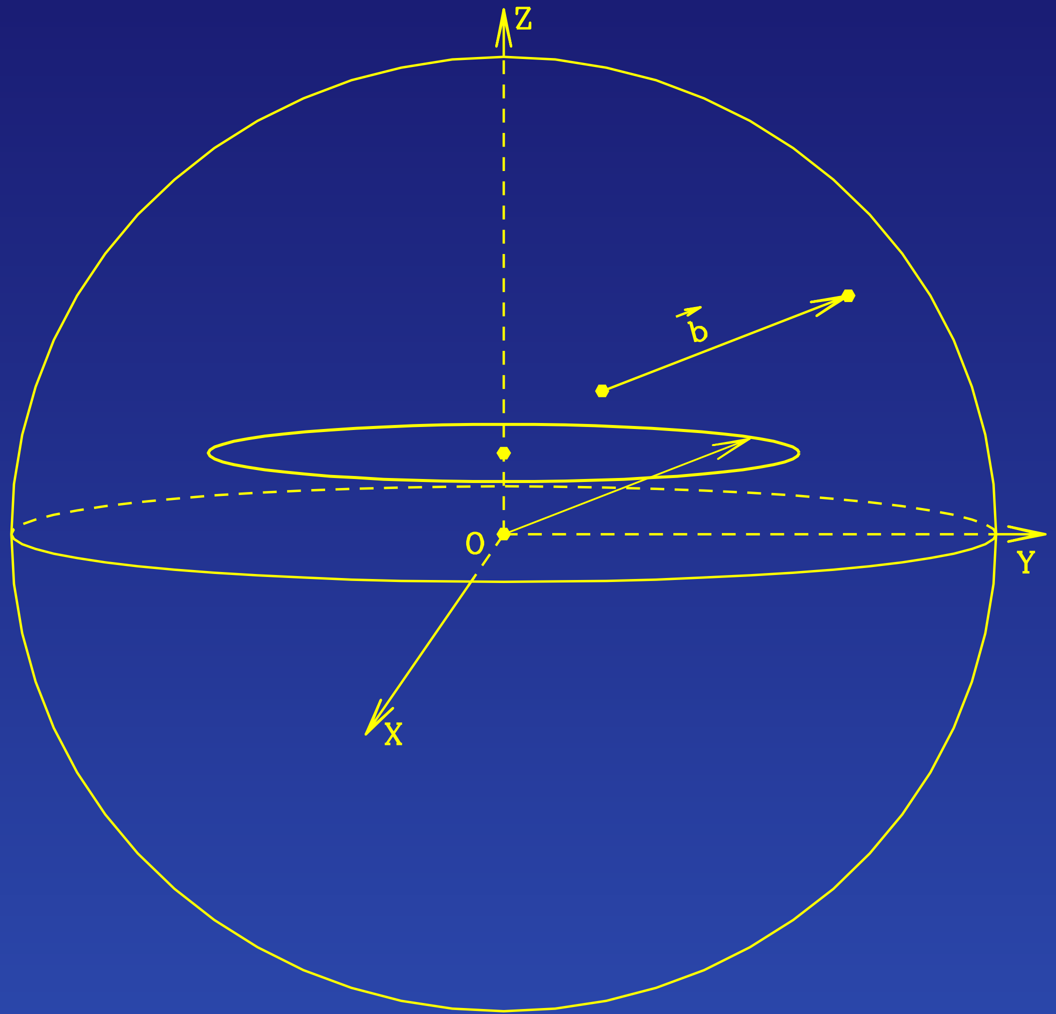
$$V \sim \exp\left(-2\pi i\nu\tau\right), \; \tau \approx \frac{\left(\overrightarrow{s}\,\overrightarrow{B}\right)}{c}$$

- Phase-tracking interferometer

$$V \sim \exp\left(-2\pi i\nu\Delta\tau\right), \; \Delta\tau \approx \frac{\left(\overrightarrow{\delta s}\,\overrightarrow{B}\right)}{c}$$

- Convenient coordinate system

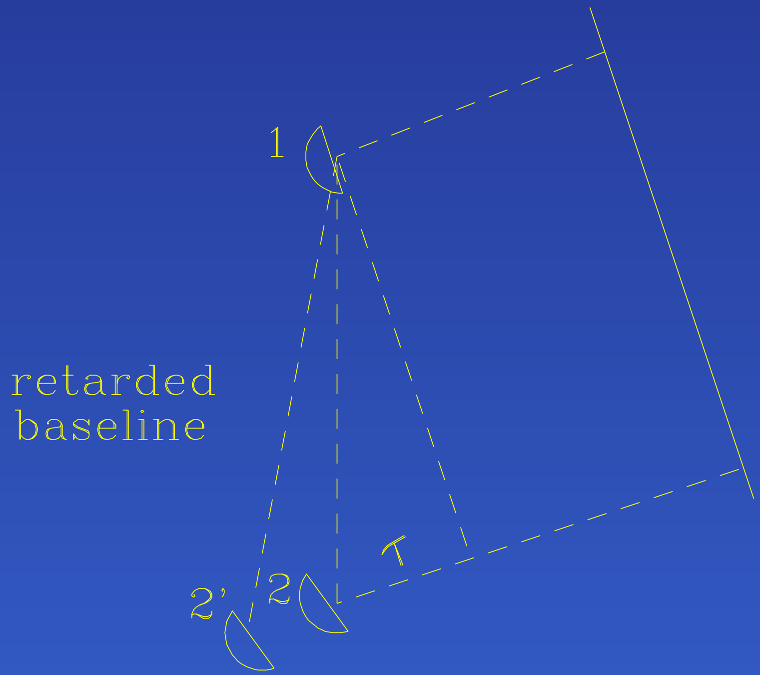
$$\overrightarrow{B} = \overrightarrow{(u, v, w)}, \; \overrightarrow{\delta s} = \overrightarrow{\left(l, m, \sqrt{1 - l^2 - m^2} - 1\right)}$$



- A very simple model is used in most software simulators to calculate the spatial frequencies probed by each baseline
- A baseline vector $\vec{b} = \overrightarrow{(L_X, L_Y, L_Z)}$ is transformed to a different coordinate system (u, v, w) for a given hour angle H and declination δ
- However, an inertial reference frame is required!

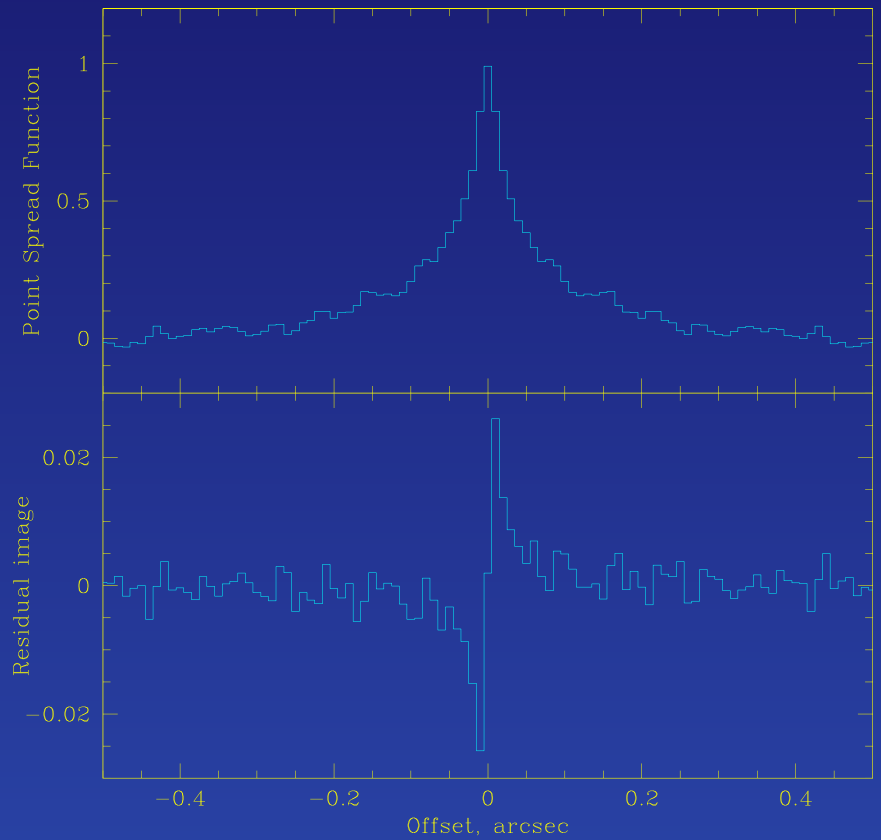
$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H & \cos H & 0 \\ -\sin\delta\cos H & \sin\delta\sin H & \cos\delta \\ \cos\delta\cos H & -\cos\delta\sin H & \sin\delta \end{pmatrix} \begin{pmatrix} L_X \\ L_Y \\ L_Z \end{pmatrix}$$

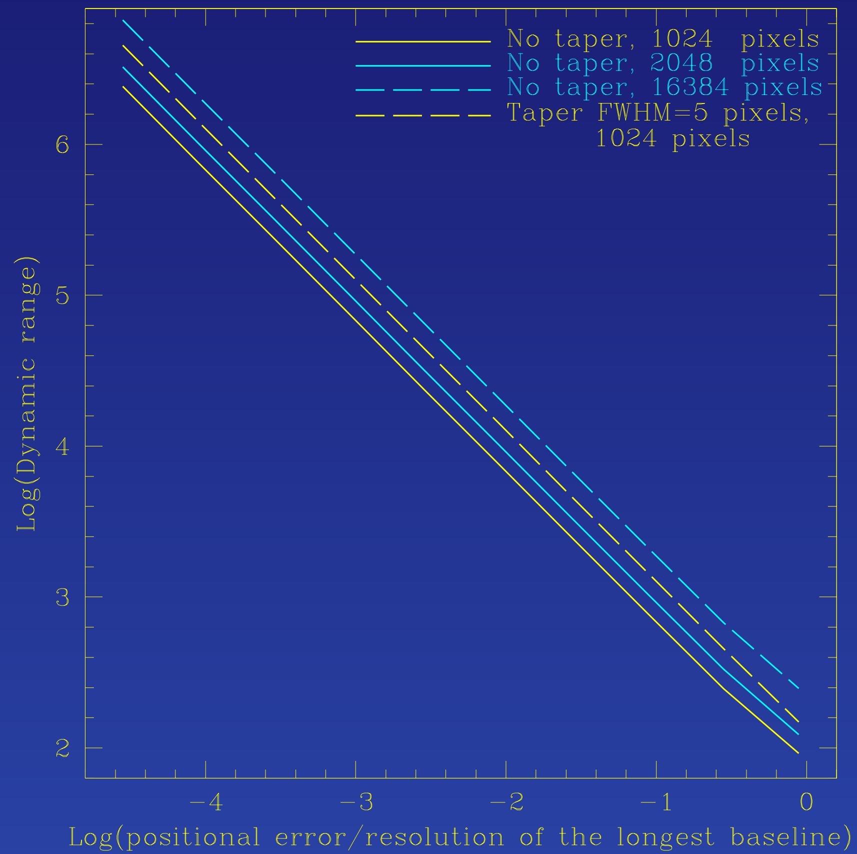
- VLBI Astrometry: a direct modelling of the geometric delay
  - Baseline length is calculated in the barycentric frame, which is assumed to be inertial
  - Weak effects like the gravitational delay, the length contraction, and the time retardation are taken into account
- A typical measurement accuracy is about 30–50 ps. Current relativistic VLBI theory provide an accuracy of about 1 ps (see e.g., Kaplan 1998, ApJ, 115, 361; Soffel et al. 1991, AJ, 101, 2306).
- Good thing: only differential effects can affect the dynamic range



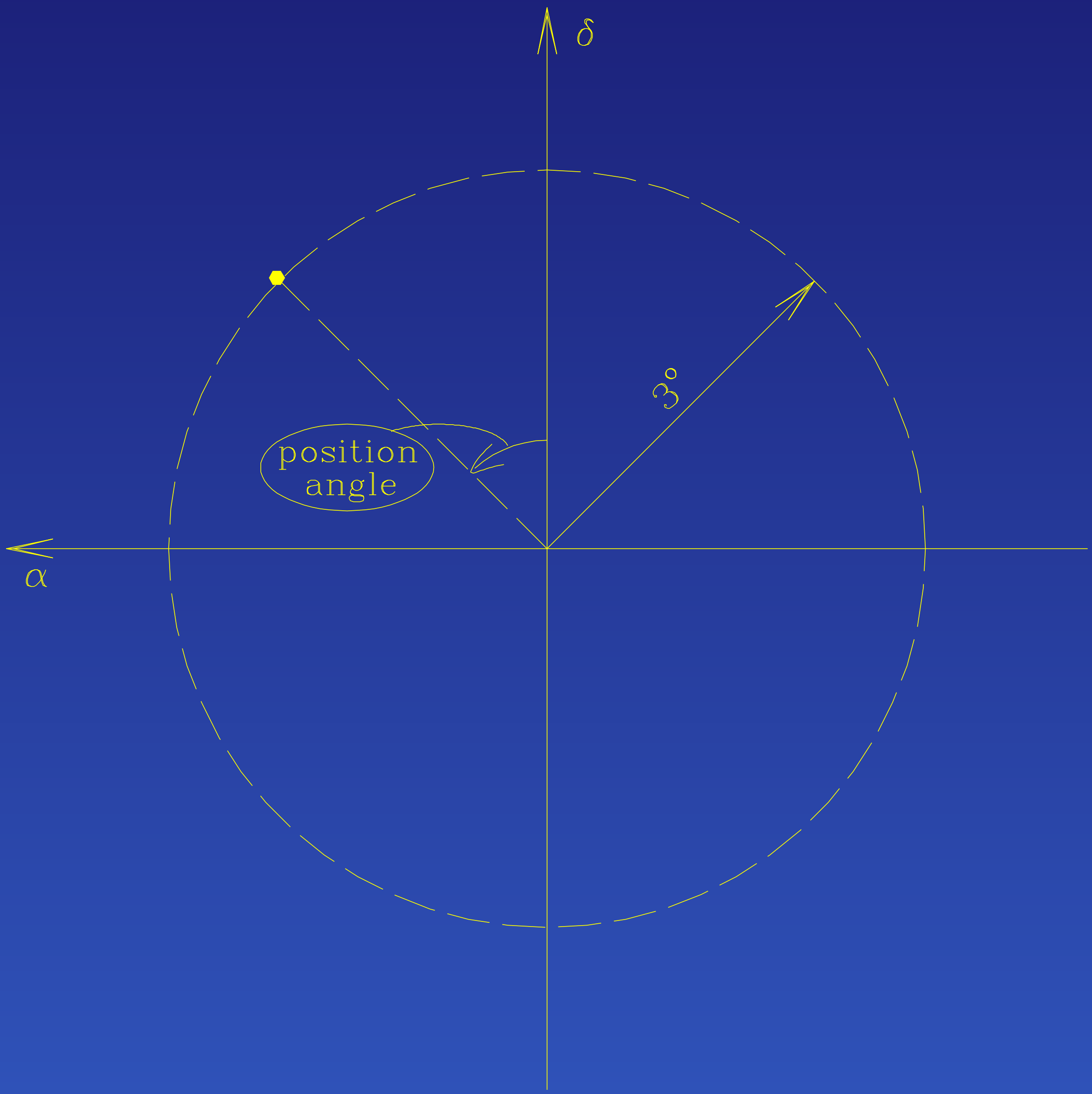
Non-inertiality produces:

- Time-average smearing
  - Not a problem
- Retarded baseline effect
- Differential Doppler shift





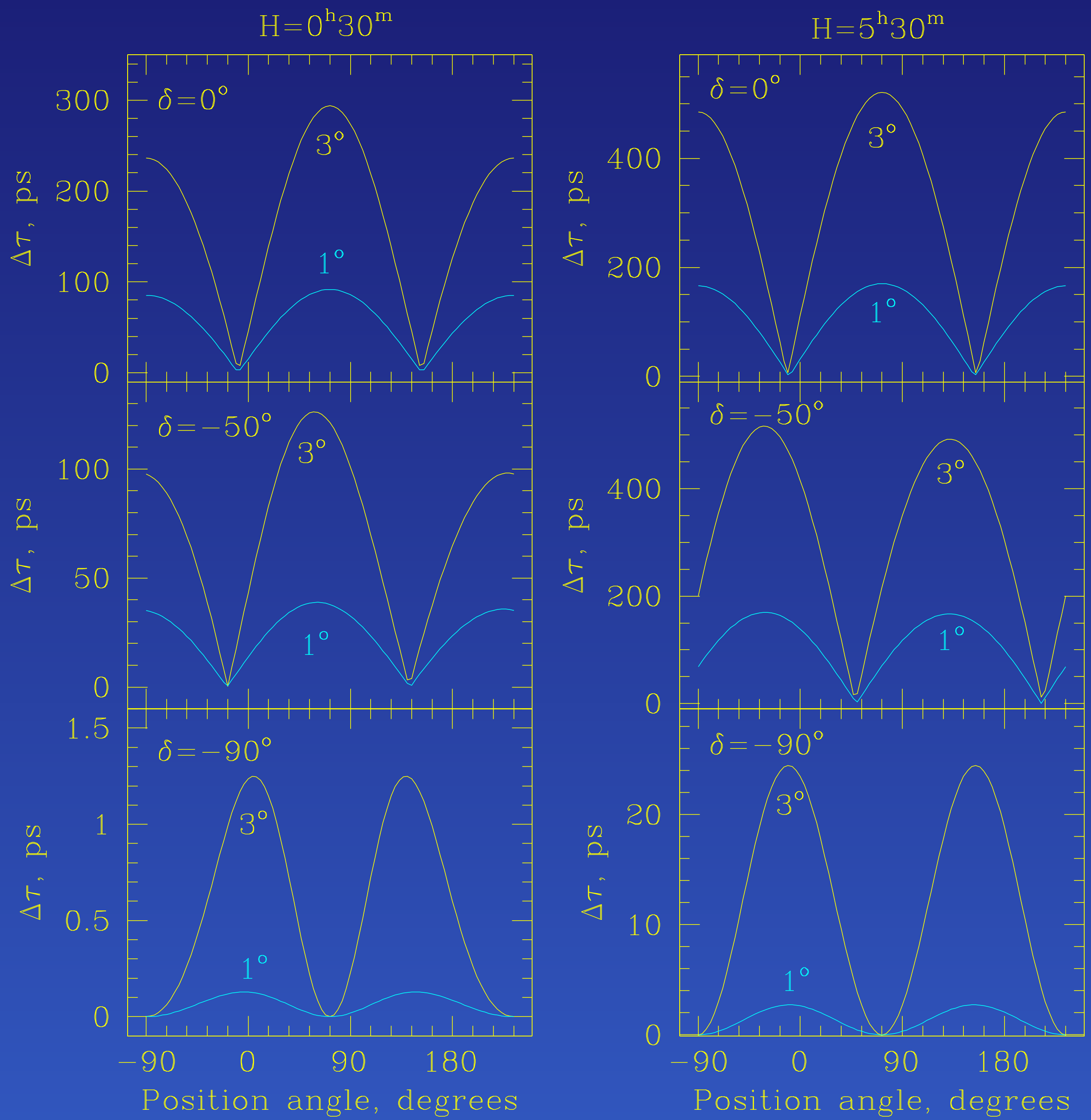
- Due to positional error the residual emission does not vanish
- If a dynamic range as high as 10^6 is required, unaccounted baseline-dependent or time-dependent delays in the field of view should be less than 0.1 ps



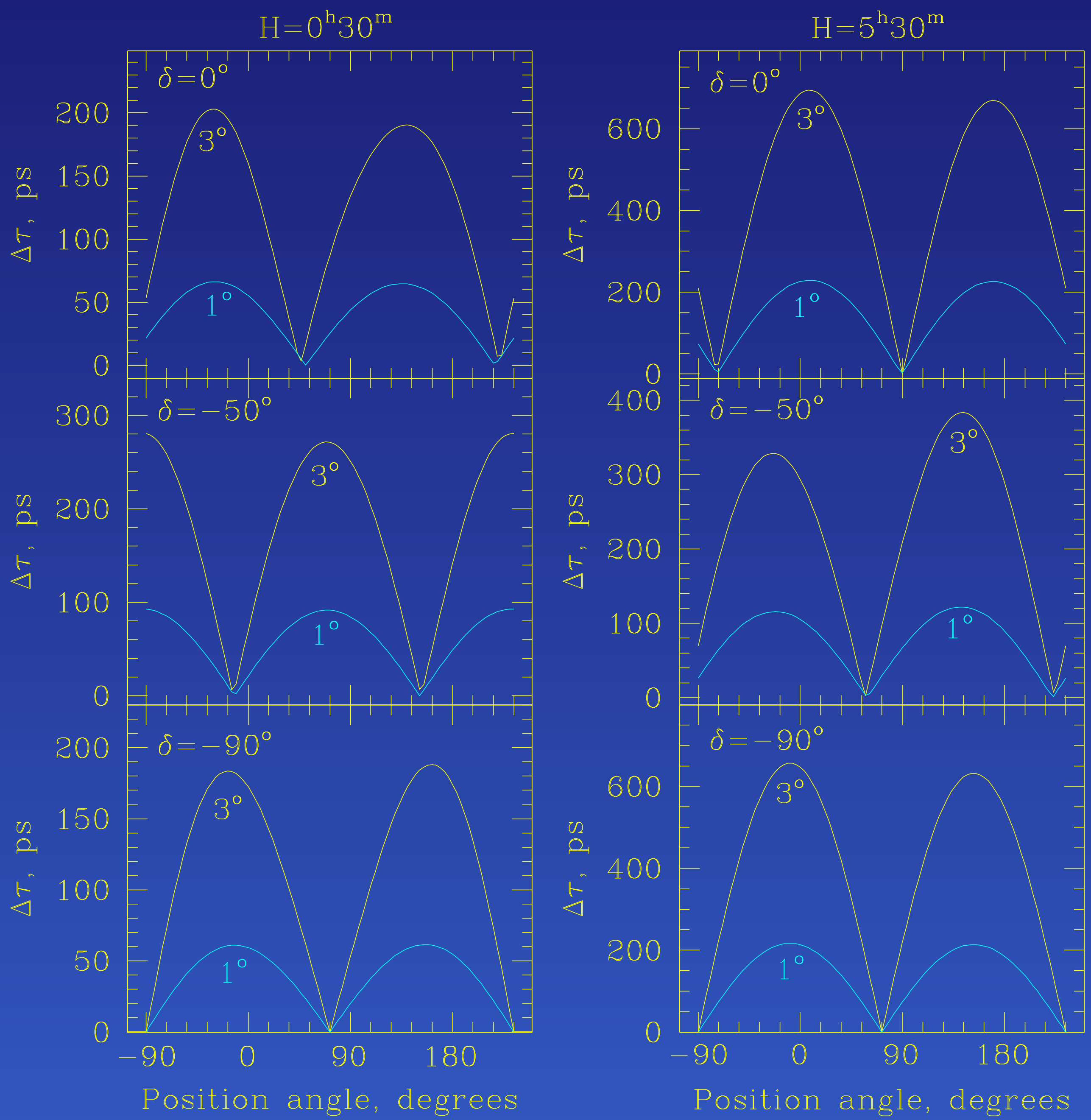
- First order estimate, semi-classical derivation

$$\tau_1 = \frac{\left(\overrightarrow{s_0}+\overrightarrow{\delta s}\right)\overrightarrow{B}\left(\overrightarrow{\delta s}\,\dot{\overrightarrow{r_2}}\right)}{c^2}$$

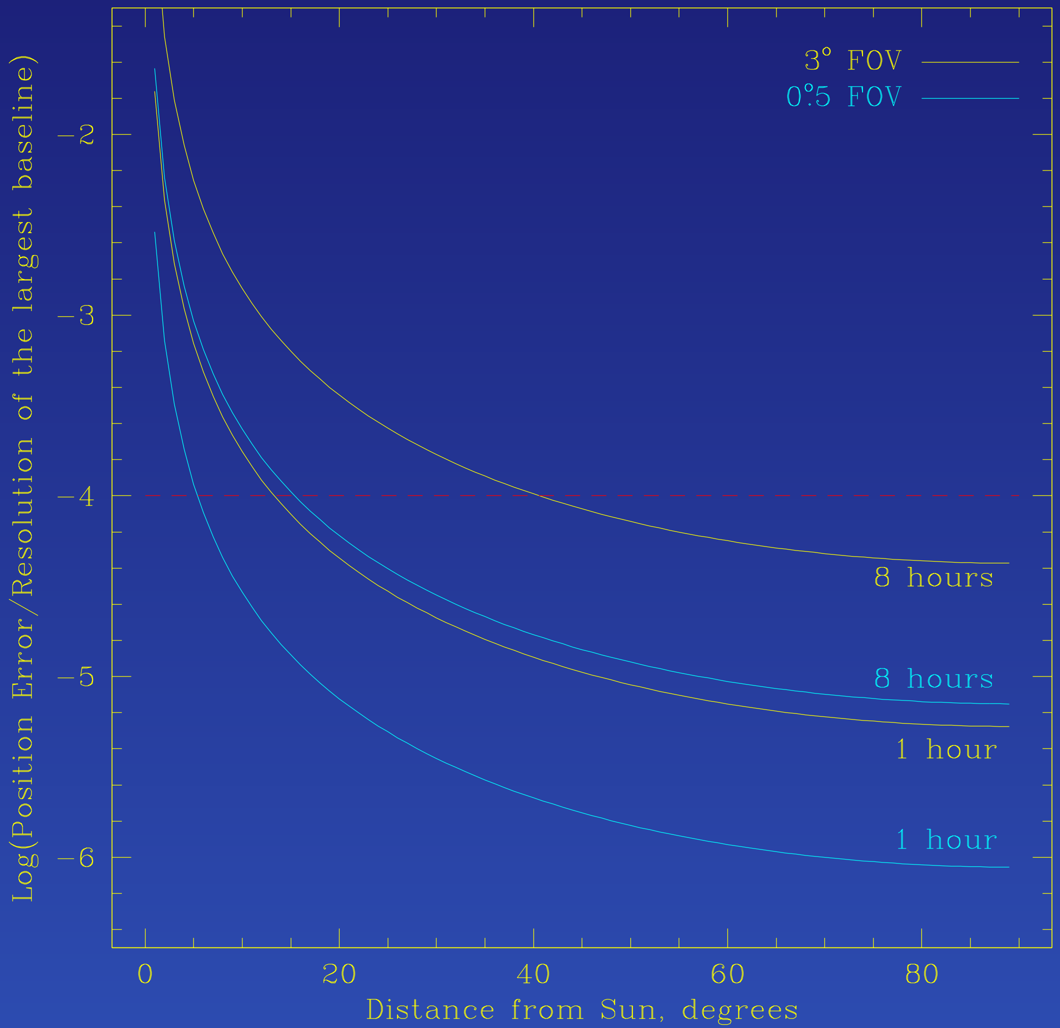
- An additional delay caused by retarded baselines depends on the position in the image and the baseline
- There is a position angle where the effect is small



- The figure shows an additional delay at 1^o and 3^o offsets from the phase centre versus the position angle
- East-West 3000 km baseline



- The figure shows an additional delay at 1^o and 3^o offsets from the phase centre versus the position angle
- North-South 3000 km baseline



- Differential gravitational bending $\rightarrow$ time-dependent distortion
- First order estimate for position accuracy, short observations ($\ll 1$ year)

$$\varepsilon = \frac{2(1+\gamma_{ppn})GM_{\odot}\Delta\phi}{c^2 R \sin^2\phi_0}\dot{\phi}_0 t_{obs},$$

ϕ_0, $\dot{\phi}_0$ – the angular separation between the phase centre and the Sun and its rate

$\Delta\phi$ – the source distance from the phase centre

γ_{ppn} – post-Newtonian parameter (=1 in GR)

Summary

- Retarded baseline effect is able to limit the dynamic range. For 1.4 GHz observations with 3000 km baselines, a dynamic range of about 10^6 can be achieved for a field of view as narrow as 12".
- Differential gravitational bending may also be important for long observations
- These limiting factors are not fundamental because algorithms dealing with the w-term can be modified to solve the problem. However, this is an additional complication for the software.
- Differential Doppler shift is unlikely to be a problem
  - Quadratic dependence on the offset $\rightarrow$ small magnitude ($< 2 \times 10^{-3}$ km s^{-1} at 3^o)
  - May be important for very narrow spectral line observations, where ultra-high dynamic range is not required