Non-Inertial Reference Frame and Dynamic Range Limit

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(a part of the bigger project conducted in collaboration with Mark Wieringa and Tim Cornwell)

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Basics of interferometry:

- Monochromatic plane wave, single polarization, no primary beam effects

\[ V \sim \exp(-2\pi i \nu \tau), \quad \tau \approx \frac{\left< \hat{s} \hat{B} \right>}{c} \]

- Phase-tracking interferometer

\[ V \sim \exp(-2\pi i \nu \Delta \tau), \quad \Delta \tau \approx \frac{\left< \hat{s} \delta \hat{B} \right>}{c} \]

- Convenient coordinate system

\[ \vec{B} = (u, v, w), \quad \delta \hat{s} = \left( l, m, \sqrt{1 - l^2 - m^2} - 1 \right) \]
A very simple model is used in most software simulators to calculate the spatial frequencies probed by each baseline.

A baseline vector \( \hat{b} = (L_X, L_Y, L_Z) \) is transformed to a different coordinate system \((u, v, w)\) for a given hour angle \(H\) and declination \(\delta\).

However, an inertial reference frame is required!

\[
\begin{pmatrix}
u \\ v \\ w
\end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix}
sin H & \cos H & 0 \\ -\sin \delta \cos H & \sin \delta \sin H & \cos \delta \\ \cos \delta \cos H & -\cos \delta \sin H & \sin \delta
\end{pmatrix} \begin{pmatrix}
L_X \\ L_Y \\ L_Z
\end{pmatrix}
\]
VLBI Astrometry: a direct modelling of the geometric delay

- Baseline length is calculated in the barycentric frame, which is assumed to be inertial
- Weak effects like the gravitational delay, the length contraction, and the time retardation are taken into account
- Good thing: only differential effects can affect the dynamic range

Non-inertiality produces:

- Time-average smearing
  - Not a problem
- Retarded baseline effect
- Differential Doppler shift
Due to positional error the residual emission does not vanish.

If a dynamic range as high as \(10^6\) is required, unaccounted baseline-dependent or time-dependent delays in the field of view should be less than 0.1 ps.
- First order estimate, semi-classical derivation

\[
\tau_1 = \frac{(s_0 + \delta s) \vec{B} \left( \delta s \vec{r}_2 \right)}{c^2}
\]

- An additional delay caused by retarded baselines depends on the position in the image and the baseline

- There is a position angle where the effect is small
The figure shows an additional delay at $1^\circ$ and $3^\circ$ offsets from the phase centre versus the position angle.

East-West 3000 km baseline
The figure shows an additional delay at $1^\circ$ and $3^\circ$ offsets from the phase centre versus the position angle.

North-South 3000 km baseline
- Differential gravitational bending → time-dependent distortion

- First order estimate for position accuracy, short observations ($\ll 1$ year)

$$\varepsilon = \frac{2(1+\gamma_{ppn})GM_\odot \Delta \phi}{c^2 R \sin^2 \phi_0} \dot{\phi}_0 t_{\text{obs}},$$

$\phi_0, \dot{\phi}_0$ – the angular separation between the phase centre and the Sun and its rate

$\Delta \phi$ – the source distance from the phase centre

$\gamma_{ppn}$ – post-Newtonian parameter (=1 in GR)
Summary

- Retarded baseline effect is able to limit the dynamic range. For 1.4 GHz observations with 3000 km baselines, a dynamic range of about $10^6$ can be achieved for a field of view as narrow as 12’’.

- Differential gravitational bending may also be important for long observations.

- These limiting factors are not fundamental because algorithms dealing with the $w$-term can be modified to solve the problem. However, this is an additional complication for the software.

- Differential Doppler shift is unlikely to be a problem.
  - Quadratic dependence on the offset $\rightarrow$ small magnitude ($< 2 \times 10^{-3}$ km s$^{-1}$ at 3°)
  - May be important for very narrow spectral line observations, where ultra-high dynamic range is not required.